Doppler Signal Detection with Negative-Resistance Diode Oscillators

YOICHIRO TAKAYAMA

Abstract—A theory is presented for the Doppler signal detection with a negative-resistance diode oscillator operating simultaneously as a signal source and Doppler signal detector. The theory is based on a realistic model of the oscillator, including an object passing in front of an antenna, and includes the previous treatments [1], [2] as the limiting cases. The effect of the bias circuit taking out the Doppler signal on the RF operation of the oscillator is taken into account self-consistently. The frequency down-conversion with a free-running oscillator is also investigated. Conversion gain is demonstrated by the experiment using a Gunn oscillator with a movable load.

I. Introduction

It has been recognized that microwave negative-resistance diodes, such as Gunn diodes and IMPATT diodes, simultaneously exhibit oscillation and nonlinear mixing [1]–[8]. Doppler radar equipment, such as Doppler speed meters and intruder alarms [5], [6], [8], are typical applications of Doppler signal detection with Gunn or IMPATT oscillators which operate simultaneously as a signal source and signal detector.

Analysis of an oscillator as a Doppler signal detector has been presented by Nagano *et al.* [1] and Mitsui *et al.* [2]. Nagano *et al.* described the effect of the load motion on the oscillator in the static condition in the analysis of the load variation detector. Mitsui *et al.* [2] analyzed the Doppler signal detection with the Gunn oscillator as the injection locking by a reflected wave with a Doppler shifted frequency.

The purpose of this paper is to present a more general theory of the negative-resistance diode oscillator simultaneously as a signal source and detector of a Doppler signal reflected by a moving object. The analysis includes the previous treatments [1], [2] as the limiting cases. The effect of the diode bias circuit taking out the Doppler signal on the RF operation of the oscillator is taken into account self-consistently. The down-conversion with a free-running oscillator is also investigated. Conversion gain is demonstrated by the experiment using a Gunn oscillator with a movable load.

II. Admittance of a Moving Object

Fig. 1 shows the basic network of the oscillator, including the moving object. The active network (whose admittance is Y_1) is connected to the ideal matched antenna through the lossless transmission line with characteristic admittance Y_0.

In this section, the admittance was set up looking toward a moving object through the antenna. The simplifying assumption was made of a perfect conductor moving along the x axis with velocity v. The effective reflection coefficient is introduced looking toward the moving object through the antenna at reference plane A, and its magnitude Γ is assumed to be constant.[1]

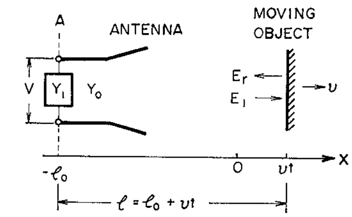

Fig. 1. Diagram for an oscillator including a moving object.

Assume that the conductor lies in the plane $x=0$ at $t=0$ and that a plane wave

$$E_i = e^{j\omega(t-(x/c))}$$

is incident on it. Then the reflected wave is [9], [10]

$$E_r = e^{j\omega_r(t+(x/c))}$$

where

$$\omega_r = \frac{1-(v/c)}{1+(v/c)}\,\omega. \tag{1}$$

Hence we obtain the effective reflection coefficient at reference plane A in $x=-l_0$ by

$$\Gamma_L = \Gamma e^{j\{(\omega_r-\omega)t-(\omega_r+\omega)(l_0/c)\}}. \tag{2}$$

Concentrate on the case where $v/c \ll 1$. In this case, we have

$$\omega_r - \omega = -\frac{2(v/c)}{1+(v/c)}\,\omega \simeq -2\frac{v}{c}\,\omega = \Delta\omega \tag{3}$$

$$\omega_r + \omega = \frac{2\omega}{1+(v/c)} \simeq 2\omega. \tag{4}$$

Using (3) and (4), we obtain

$$\Gamma_L \simeq \Gamma e^{-j(2\omega/c)(l_0+vt)} = \Gamma e^{j(\Delta\omega t-\theta_0)} = \Gamma e^{-j2\beta l} \tag{5}$$

where $\beta=\omega/c$, $\theta_0=2\beta l_0$, and $l=l_0+vt$ is the distance between reference plane A and the moving object at time t.

From (5), the admittance looking toward the moving object at reference plane A is

$$Y_L = \frac{1-\Gamma_L}{1+\Gamma_L}\,Y_0 = G_L + jB_L$$

$$G_L = \frac{1-\Gamma^2}{1+\Gamma^2+2\Gamma\cos 2\beta l}\,Y_0$$

$$B_L = \frac{2\Gamma\sin 2\beta l}{1+\Gamma^2+2\Gamma\cos 2\beta l}\,Y_0. \tag{6}$$

Manuscript received June 15, 1972; revised July 31, 1972.

The author is with the Central Research Laboratories, Nippon Electric Co., Ltd., Kawasaki, Japan.

[1] Γ^2 is the ratio of the received power to the radiated power through the antenna.

If $\Gamma \ll 1$, we have

$$G_L \simeq Y_0 + \Delta G_L \qquad B_L \simeq \Delta B_L$$
$$\Delta G_L = -2Y_0\Gamma \cos 2\beta l \qquad \Delta B_L = 2Y_0\Gamma \sin 2\beta l. \tag{7}$$

Equation (6) or (7) is the admittance looking toward a moving object through the matched antenna.

III. Basic Oscillator Equations, Including Moving Object

The oscillator is investigated, including a moving object, as shown in Fig. 1. In this paper, a dc bias-voltage controlled device, such as a Gunn diode, is described. The bias circuit taking out the Doppler signal is included in the analysis. It is assumed that the device admittance Y_1 is a function of operating frequency ω, ac voltage amplitude $\tilde{V}$ at the reference plane, and dc or slowly varying[2] bias voltage V_B.

Corresponding to the condition that the total admittance of the oscillator is equal to zero,

$$Y = Y_1(\omega, \tilde{V}, V_B) + Y_L(\omega, t) = 0. \tag{8}$$

The ac voltage V across Y_1, which is sinusoidal with slowly varying amplitude and phase, can be expressed by

$$V = \tilde{V}(t)e^{j(\omega_0 t + \theta(t))}. \tag{9}$$

In (8), it is assumed that $\tilde{V}$, ω, and V_B are deviated slightly from the steady-state values $\tilde{V}_0$, ω_0, and V_{B_0}, respectively. Then the deviation of the admittance Y can be expressed as

$$\delta Y = \left(\frac{\partial Y}{\partial \omega}\right)_{\omega_0} \cdot (\omega - \omega_0) + \left(\frac{\partial Y}{\partial \tilde{V}}\right)_{\tilde{V}_0} \cdot (\tilde{V} - \tilde{V}_0) + \left(\frac{\partial Y}{\partial V_B}\right)_{V_{B_0}} \cdot (V_B - V_{B_0}). \tag{10}$$

From (8), we have

$$Y_L(\omega_0, t) + Y_1(\omega_0, \tilde{V}_0, V_{B_0}) + \delta Y = 0. \tag{11}$$

Replacing ω by $\omega_0 + (d\theta/dt) - j(1/\tilde{V})\cdot(d\tilde{V}/dt)$ (see [11], [12]),

$$Y_L(\omega_0, t) + Y_1(\omega_0, \tilde{V}_0, V_{B_0}) + \left(\frac{\partial Y}{\partial \omega}\right)\left(\frac{d\theta}{dt} - j\,\frac{1}{\tilde{V}}\cdot\frac{d\delta\tilde{V}}{dt}\right) + \left(\frac{\partial Y}{\partial \tilde{V}}\right)\delta\tilde{V} + \left(\frac{\partial Y}{\partial V_B}\right)\delta \tilde{V}_B = 0 \tag{12}$$

where

$$\delta\tilde{V} = \tilde{V} - \tilde{V}_0 \quad \delta V_B = V_B - V_{B_0}$$
$$Y_L(\omega_0, t=0) = Y_0 \quad Y_L(\omega_0, t=0) + Y_1(\omega_0, \tilde{V}_0, V_{B_0}) = 0$$

and ω_0, $\tilde{V}_0$, V_{B_0}, and Y_0 are the oscillating frequency, the ac terminal voltage amplitude, the dc bias voltage, and the load conductance, respectively, with no moving object.

From (7) and (8)

$$Y_L(\omega_0, t) + Y_1(\omega_0, \tilde{V}_0, V_{B_0}) = \Delta G_L + j\Delta B_L. \tag{13}$$

[2] For $\Delta\omega$, at which this assumption is justifiable, the present theory is valid.

Substituting (13) into (12), we obtain

$$\left(\frac{\partial G}{\partial \omega}\right)\frac{d\theta}{dt} + \left(\frac{\partial B}{\partial \omega}\right)\frac{1}{\tilde{V}}\cdot\frac{d\delta\tilde{V}}{dt} + \left(\frac{\partial G}{\partial \tilde{V}}\right)\delta\tilde{V} + \left(\frac{\partial G}{\partial V_B}\right)\delta V_B + \Delta G_L = 0 \tag{14a}$$

$$\left(\frac{\partial B}{\partial \omega}\right)\frac{d\theta}{dt} - \left(\frac{\partial G}{\partial \omega}\right)\frac{1}{\tilde{V}}\cdot\frac{d\delta\tilde{V}}{dt} + \left(\frac{\partial B}{\partial \tilde{V}}\right)\delta\tilde{V} + \left(\frac{\partial B}{\partial V_B}\right)\delta V_B + \Delta B_L = 0 \tag{14b}$$

where $Y = G + jB$.

It is useful here to define the parameters

$$P_{ex,0} = \frac{\omega_0}{2Y_0}\left(\frac{\partial G_1}{\partial \omega}\right)_{\omega_0}$$
$$P_{ex,L} = \frac{\omega_0}{2Y_0}\left(\frac{\partial G_L}{\partial \omega}\right)_{\omega_0} = 2\Gamma\beta l \sin 2\beta l$$
$$Q_{ex,0} = \frac{\omega_0}{2Y_0}\left(\frac{\partial B_1}{\partial \omega}\right)_{\omega_0}$$
$$Q_{ex,L} = \frac{\omega_0}{2Y_0}\left(\frac{\partial B_L}{\partial \omega}\right)_{\omega_0} = 2\Gamma\beta l \cos 2\beta l$$
$$P_{ex} = P_{ex,0} + P_{ex,L}$$
$$Q_{ex} = Q_{ex,0} + Q_{ex,L} \tag{15}$$

where $Y_1 = G_1 + jB_1$, and $Q_{ex,L}$ and $P_{ex,L}$ are functions of time t, derived from (7). Introducing these parameters into (14a) and (14b), and using $\partial Y_L/\partial\tilde{V} = \partial Y_L/\partial V_B = 0$, we obtain

$$P_{ex}\frac{d\theta}{dt} + Q_{ex}\frac{1}{\tilde{V}}\cdot\frac{d\delta\tilde{V}}{dt} + \frac{\omega_0}{2Y_0}\left(\frac{\partial G_1}{\partial \tilde{V}}\right)\delta\tilde{V} + \frac{\omega_0}{2Y_0}\left(\frac{\partial G_1}{\partial V_B}\right)\delta V_B + \frac{\omega_0 \Delta G_L}{2Y_0} = 0 \tag{16a}$$

$$Q_{ex}\frac{d\theta}{dt} - P_{ex}\frac{1}{\tilde{V}}\cdot\frac{d\delta\tilde{V}}{dt} + \frac{\omega_0}{2Y_0}\left(\frac{\partial B_1}{\partial \tilde{V}}\right)\delta\tilde{V} + \frac{\omega_0}{2Y_0}\left(\frac{\partial B_1}{\partial V_B}\right)\delta V_B + \frac{\omega_0 \Delta B_L}{2Y_0} = 0. \tag{16b}$$

Equations (16a) and (16b) are the basic equations of the oscillator, including a moving object in front of an antenna.

Let us now consider the bias circuit. The dc bias current I of the dc bias-voltage controlled device, such as a Gunn diode, is a function of RF operating frequency ω, ac voltage amplitude $\tilde{V}$, and dc or slowly varying bias voltage V_B. Assume that $\tilde{V}$, ω, and V_B are deviated slightly from the steady-state values $\tilde{V}_0$, ω_0, and V_{B_0}, respectively. Then we can express the deviation of the dc bias current I as

$$\delta I = \left(\frac{\partial I}{\partial \omega}\right)_{\omega_0} \cdot \frac{d\theta}{dt} + \left(\frac{\partial I}{\partial \tilde{V}}\right)_{\tilde{V}_0} \cdot \delta\tilde{V} + \left(\frac{\partial I}{\partial V_B}\right)_{V_{B_0}} \cdot \delta V_B \tag{17}$$

where $d\theta/dt$ is the frequency deviation.

Let the conductance of the bias circuit taking out the Doppler signal be G_B. Then $\delta I = -G_B \delta V_B$. Substituting this relation into (17), we obtain

$$\delta V_B = -\frac{\left(\frac{\partial I}{\partial \omega}\right)\cdot \frac{d\theta}{dt} + \left(\frac{\partial I}{\partial \tilde{V}}\right)\cdot \delta \tilde{V}}{G_B + \left(\frac{\partial I}{\partial V_B}\right)}. \quad (18)$$

Substituting (18) into (16) and eliminating δV_B, we have

$$P\frac{d\theta}{dt} + Q_{ex}\frac{1}{\tilde{V}}\cdot\frac{d\delta\tilde{V}}{dt} + \frac{\omega_0}{\tilde{V}}\cdot J\delta\tilde{V} + \frac{\omega_0 \Delta G_L}{2Y_0} = 0 \quad (19a)$$

$$Q\frac{d\theta}{dt} - P_{ex}\frac{1}{\tilde{V}}\cdot\frac{d\delta\tilde{V}}{dt} + \frac{\omega_0}{\tilde{V}}\cdot K\delta\tilde{V} + \frac{\omega_0 \Delta B_L}{2Y_0} = 0 \quad (19b)$$

where

$$P = P_{ex} - \frac{\frac{\omega_0}{2Y_0}\left(\frac{\partial G_1}{\partial V_B}\right)\left(\frac{\partial I}{\partial \omega}\right)}{G_B + \left(\frac{\partial I}{\partial V_B}\right)} \quad (20a)$$

$$Q = Q_{ex} - \frac{\frac{\omega_0}{2Y_0}\left(\frac{\partial B_1}{\partial V_B}\right)\left(\frac{\partial I}{\partial \omega}\right)}{G_B + \left(\frac{\partial I}{\partial V_B}\right)} \quad (20b)$$

$$J = \frac{\tilde{V}_0}{2Y_0}\left(\frac{\partial G_1}{\partial \tilde{V}}\right) - \frac{\frac{\tilde{V}_0}{2Y_0}\left(\frac{\partial G_1}{\partial V_B}\right)\left(\frac{\partial I}{\partial \tilde{V}}\right)}{G_B + \left(\frac{\partial I}{\partial V_B}\right)} \quad (21a)$$

$$K = \frac{\tilde{V}_0}{2Y_0}\left(\frac{\partial B_1}{\partial \tilde{V}}\right) - \frac{\frac{\tilde{V}_0}{2Y_0}\left(\frac{\partial B_1}{\partial V_B}\right)\left(\frac{\partial I}{\partial \tilde{V}}\right)}{G_B + \left(\frac{\partial I}{\partial V_B}\right)}. \quad (21b)$$

Equations (19a) and (19b) describe completely the RF behavior of the oscillator including the moving object. In these equations, the bias circuit taking out δV_B (which is the detected Doppler signal) is taken into account.

Note that dc bias-current controlled devices, such as IMPATT diodes, can also be described similarly.

IV. Detailed Discussions

Eliminating $d\theta/dt$ from (19a) and (19b), we obtain

$$(PP_{ex} + QQ_{ex})\frac{1}{\tilde{V}}\cdot\frac{d\delta\tilde{V}}{dt} + \frac{\omega_0}{\tilde{V}}(JQ - KP)\delta\tilde{V} + \frac{\omega_0}{2Y_0}(Q\Delta G_L - P\Delta B_L) = 0. \quad (22)$$

Not only G_L and B_L, but also P_{ex}, Q_{ex}, P, and Q are time-dependent parameters. This makes the analytical solution of (22) difficult.

In the following, let us examine somewhat simplified, but essential cases.

A. When $P_{ex,0} \gg P_{ex,L}$, $Q_{ex,0} \gg Q_{ex,L}$

First, it is assumed that $P_{ex,0} \gg 2\Gamma\beta l$ and $Q_{ex,0} \gg 2\Gamma\beta l$. This corresponds to the case where the $Q_{ex,L}$ by the moving object can be neglected in the total external Q, Q_{ex} of the oscillator. In this case, from (22), we have

$$(P_0P_{ex,0} + Q_0Q_{ex,0})\frac{1}{\tilde{V}_0}\cdot\frac{d\delta\tilde{V}}{dt} + \frac{\omega_0}{\tilde{V}_0}(JQ_0 - KP_0)\delta\tilde{V} = \omega_0\Gamma\{Q_0\cos(\Delta\omega t - \theta_0) - P_0\sin(\Delta\omega t - \theta_0)\} \quad (23)$$

where

$$P_0 = P_{ex,0} - \frac{\frac{\omega_0}{2Y_0}\left(\frac{\partial G_1}{\partial V_B}\right)\left(\frac{\partial I}{\partial \omega}\right)}{G_B + \left(\frac{\partial I}{\partial V_B}\right)}$$

$$Q_0 = Q_{ex,0} - \frac{\frac{\omega_0}{2Y_0}\left(\frac{\partial B_1}{\partial V_B}\right)\left(\frac{\partial I}{\partial \omega}\right)}{G_B + \left(\frac{\partial I}{\partial V_B}\right)}.$$

Solving (23) and neglecting the term decaying with time, we obtain

$$\delta\tilde{V} = \frac{\Gamma\tilde{V}_0}{p^2\left(\frac{\Delta\omega}{\omega_0}\right)^2 + k^2}\left[\left\{Q_0p\left(\frac{\Delta\omega}{\omega_0}\right) - P_0k\right\}\cdot\sin(\Delta\omega t - \theta_0) + \left\{Q_0k + P_0p\left(\frac{\Delta\omega}{\omega_0}\right)\right\}\cos(\Delta\omega t - \theta_0)\right] \quad (24)$$

$$= \frac{\Gamma\tilde{V}_0\sqrt{P_0^2 + Q_0^2}}{\sqrt{p^2\left(\frac{\Delta\omega}{\omega_0}\right)^2 + k^2}}\cdot\sin(\Delta\omega t - \theta_0 + \phi) \quad (25)$$

where

$$\tan\phi = \frac{P_0p(\Delta\omega/\omega_0) + Q_0k}{Q_0p(\Delta\omega/\omega_0) - P_0k}$$

$$p = P_0P_{ex,0} + Q_0Q_{ex,0} > 0, \qquad k = JQ_0 - KP_0 > 0.$$

Substituting (24) into (19), we have

$$\frac{d\theta}{dt} = \frac{\omega_0\Gamma}{p}\left[Q_{ex,0} - \frac{h\{Q_0p(\Delta\omega/\omega_0) - P_0k\}}{p^2(\Delta\omega/\omega_0)^2 + k^2}\right]\sin(\Delta\omega t - \theta_0) + \frac{\omega_0\Gamma}{p}\left[P_{ex,0} - \frac{h\{P_0p(\Delta\omega/\omega_0) + Q_0k\}}{p^2(\Delta\omega/\omega_0)^2 + k^2}\right]\cdot\cos(\Delta\omega t - \theta_0) \quad (26)$$

where

$$h = JP_{ex,0} + KQ_{ex,0}.$$

Substituting (24) and (26) into (18), we obtain δV_B, which is the detected Doppler signal whose frequency is $\Delta\omega$.

In the case where $P_{ex,0}=P_0=0$, $K=0$, $Q_0=Q_{ex,0}$, and $J=J_0=(\tilde{V}_0/2Y_0)(\partial G_1/\partial \tilde{V})$, from (24) and (26), we have

$$\delta\tilde{V} = \frac{\Gamma V_0}{Q_{ex,0}^2\left(\frac{\Delta\omega}{\omega_0}\right)^2 + J_0^2} \cdot \left[Q_{ex,0}\left(\frac{\Delta\omega}{\omega_0}\right) \sin(\Delta\omega t - \theta_0) + J_0 \cos(\Delta\omega t - \theta_0)\right]$$

$$\frac{d\theta}{dt} = \left(\frac{\omega_0\Gamma}{Q_{ex,0}}\right) \sin(\Delta\omega t - \theta_0).$$

These expressions are the same as those obtained by Mitsui *et al.* [2]. In addition, assuming $J_0^2 \gg Q_{ex,0}^2(\Delta\omega/\omega_0)^2$, we have

$$\delta\tilde{V} = \Gamma V_0 \cos(\Delta\omega t - \theta_0)$$

$$\frac{d\theta}{dt} = \left(\frac{\omega_0\Gamma}{Q_{ex,0}}\right) \sin(\Delta\omega t - \theta_0).$$

These results coincide with the expressions obtained by Nagano *et al.* [1].

B. When $Q_{ex,0} \sim Q_{ex,L}$

Let us next consider the case where $Q_{ex,L}$ by the moving object cannot be neglected in the Q_{ex} of the oscillator. This case has not been taken into account in the previous treatments [1], [2].

From a practical point of view, the present analysis is mainly concerned with the case where $Q_{ex,L}$ or $2\Gamma\beta l$ is considerably small, relative to $Q_{ex,0}$. For simplicity, it is assumed that $Q \simeq Q_{ex}$, $P_{ex} \simeq 0$, and $K \simeq 0$. It is further assumed that $|\Delta\omega/\omega_0| Q_{ex,0} \ll J$. This assumption is justifiable in the practical device[3] (where $\omega_0 \sim 10^{10}$, $\Delta\omega \lesssim 10^5$, $Q_{ex,0} \lesssim 10^2$, and $J \sim 1$).

Then, from (19) and (20), we have

$$Q_{ex,0}\{1 + (2\Gamma\beta l_0/Q_{ex,0}) \cos(\Delta\omega t - \theta_0)\} \frac{d\delta\tilde{V}}{dt} + \omega_0 J \delta\tilde{V} = \omega_0 \Gamma \tilde{V}_0 \cos(\Delta\omega t - \theta_0) \quad (27)$$

$$\frac{d\theta}{dt} = \frac{(\Gamma/Q_{ex,0}) \sin(\Delta\omega t - \theta_0)}{1 + (2\Gamma\beta l_0/Q_{ex,0}) \cos(\Delta\omega t - \theta_0)} \quad (28)$$

where $l \simeq l_0$ and $V \simeq V_0$ are used. From (28), it can immediately be seen that the frequency deviation $d\theta/dt$ has higher harmonics of $\Delta\omega$. Rewriting (28), we obtain

$$\frac{d\theta}{dt} \simeq \left(\frac{\Gamma}{Q_{ex,0}}\right) \cdot \left\{\sin(\Delta\omega t - \theta_0) - \left(\frac{\Gamma\beta l_0}{Q_{ex,0}}\right) \sin(2\Delta\omega t - 2\theta_0)\right\}.$$

In solving (27), for small $|\alpha|$ ($\alpha = \Delta\omega Q_{ex,0}/\omega_0 J$), we will obtain $\delta\tilde{V}$ as the form of power series in α by applying the perturbation method.

Rewriting (27), we have

$$\delta\tilde{V} = \frac{\Gamma\tilde{V}_0}{J}\cos(\Delta\omega t - \theta_0) - \frac{Q_{ex,0}}{\omega_0 J}\left\{1 + \frac{2\Gamma\beta l_0}{Q_{ex,0}}\cos(\Delta\omega t - \theta_0)\right\}\frac{d\delta\tilde{V}}{dt}. \quad (29)$$

The second term of (29) is regarded as the perturbation term. The zero-order $\delta\tilde{V}$ is

$$(\delta\tilde{V})_0 = (\Gamma\tilde{V}_0/J)\cos(\Delta\omega t - \theta_0). \quad (30)$$

Substituting $d(\delta\tilde{V})_0/dt$ into (29), we obtain the modified $\delta\tilde{V}$, $(\delta\tilde{V})_1$. Repeating a similar operation, we obtain highly modified $\delta\tilde{V}$ as

$$\begin{aligned}\delta\tilde{V} = {} & x\cos(\Delta\omega t - \theta_0) + x\alpha\sin(\Delta\omega t - \theta_0) \\ & - x\alpha^2\cos(\Delta\omega t - \theta_0) + \cdots \\ & + \tfrac{1}{2}x\alpha q\sin(2\Delta\omega t - 2\theta_0) \\ & - \tfrac{3}{2}x\alpha^2 q\cos(2\Delta\omega t - 2\theta_0) + \cdots \\ & - \tfrac{1}{2}x\alpha^2 q^2\cos(3\Delta\omega t - 3\theta_0) + \cdots \end{aligned} \quad (31)$$

where

$$x = \Gamma\tilde{V}_0/J, \quad q = 2\Gamma\beta l_0/Q_{ex,0} < 1, \quad \alpha = \Delta\omega Q_{ex,0}/\omega_0 J.$$

Substituting (29) and (31) into (18), we obtain the detected signal δV_B, which has the higher harmonic components of $\Delta\omega$. If $|\alpha| \ll 1$ and $q \ll 1$, the higher harmonics can be neglected.

In the particular case of $q>1$, $Q_{ex}=Q_{ex,0}+Q_{ex,L}$ becomes negative periodically. In the period of $Q_{ex}<0$, $\delta\tilde{V}$ diverges with time and the oscillator becomes unstable. For unstable operation, the present analysis becomes invalid. However, the mode jump into the stable mode of $Q_{ex}>0$ may be expected as soon as the oscillator enters into the unstable mode of $Q_{ex}<0$. In this case, from (28), it can be expected that, following the mode jump, frequency deviation $d\theta/dt$ will monotonically increase (in the case of $v<0$) or decrease (in the case of $v>0$) until the next mode jump.

V. Frequency Conversion with Free-Running Oscillator

This section presents a theory on frequency down-conversion with a free-running oscillator. The basic network is shown in Fig. 2, where V_1 and V_2 are the incident signal and output voltage waves, respectively, at reference plane A. The active network (whose admittance is Y_1), including the negative-resistance diode, is connected to the ideal isolator through the lossless transmission line with characteristic admittance Y_0. The incident signal passes through the isolator and the output power is absorbed by the isolator. It is assumed that the oscillator is not injection-locked by V_1.

V_1 and the ac voltage V across Y_1, which is sinusoidal with slowly varying amplitude and phase, can be expressed by

$$V_1 = \tilde{V}_1 e^{j(\omega_0+\Delta\omega)t} \quad (32)$$

$$V = \tilde{V}(t) e^{j(\omega_0 t+\theta(t))} \quad (33)$$

where $V=V_1+V_2$ and ω_0 is the free-running frequency. V_1 is independent of V_2. V_1 and V satisfy the relation

$$\frac{V_1}{V} = \frac{Y_0+Y_1}{2Y_0}. \quad (34)$$

[3] Note that J and K are extended definitions of nonlinearity saturation factors.

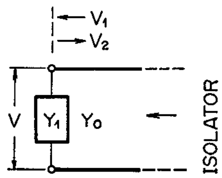

Fig. 2. Diagram for a free-running oscillator as a frequency down-converter.

In (34), assume that ω, $\tilde{V}$, and dc bias voltage V_B are deviated slightly from free-running values ω_0, V_0, and V_{B0}, respectively. Then the deviation of Y_1 can be expressed as

$$\delta Y_1 = \left(\frac{\partial Y_1}{\partial \omega}\right)_{\omega_0} \cdot (\omega - \omega_0) + \left(\frac{\partial Y_1}{\partial \tilde{V}}\right)_{\tilde{V}_0} \cdot (\tilde{V} - \tilde{V}_0) + \left(\frac{\partial Y_1}{\partial V_B}\right)_{V_{B_0}} \cdot (V_B - V_{B_0}). \quad (35)$$

From (34), we have

$$V_1 = \frac{1}{2Y_0}\left[Y_0 + Y_1(\omega_0, \tilde{V}_0, V_{B_0}) + \delta Y_1\right] V.$$

Using $Y_0 + Y_1(\omega_0, \tilde{V}_0, V_{B_0}) = 0$ and replacing ω by $\omega_1 + (d\theta/dt) - j(1/\tilde{V}) \cdot (d\tilde{V}/dt)$ in (35),

$$\frac{\tilde{V}_1}{\tilde{V}} e^{j(\Delta\omega t - \theta)} = \frac{1}{2Y_0}\left\{\left(\frac{\partial Y_1}{\partial \omega}\right)\left(\frac{d\theta}{dt} - j\frac{1}{\tilde{V}} \cdot \frac{d\delta\tilde{V}}{dt}\right) + \left(\frac{\partial Y_1}{\partial \tilde{V}}\right) \cdot \delta\tilde{V} + \left(\frac{\partial Y_1}{\partial V_B}\right)\delta V_B\right\} \quad (36)$$

where $\delta\tilde{V} = \tilde{V} - \tilde{V}_0$, $\delta V_B = V_B - V_{B_0}$. From (36), we can derive

$$\left(\frac{\partial G_1}{\partial \omega}\right) \cdot \frac{d\theta}{dt} + \left(\frac{\partial B_1}{\partial \omega}\right)\frac{1}{\tilde{V}_0} \cdot \frac{d\delta\tilde{V}}{dt} + \left(\frac{\partial G_1}{\partial \tilde{V}}\right)\delta\tilde{V} + \left(\frac{\partial G_1}{\partial V_B}\right)\delta V_B = 2Y_0\left(\frac{\tilde{V}_1}{\tilde{V}_0}\right)\cos(\Delta\omega t - \theta) \quad (37a)$$

$$\left(\frac{\partial B_1}{\partial \omega}\right) \cdot \frac{d\theta}{dt} - \left(\frac{\partial G_1}{\partial \omega}\right)\frac{1}{\tilde{V}_0} \cdot \frac{d\delta\tilde{V}}{dt} + \left(\frac{\partial B_1}{\partial \tilde{V}}\right)\delta\tilde{V} + \left(\frac{\partial B_1}{\partial V_B}\right)\delta V_B = 2Y_0\left(\frac{\tilde{V}_1}{\tilde{V}_0}\right)\sin(\Delta\omega t - \theta) \quad (37b)$$

where $Y_1 = G_1 + jB_1$ and $\tilde{V} \simeq \tilde{V}_0$. Substituting (18) into (37a) and (37b) and using (15), we obtain

$$P_0\frac{d\theta}{dt} + Q_{ex,0}\frac{1}{\tilde{V}_0} \cdot \frac{d\delta\tilde{V}}{dt} + \frac{\omega_0}{\tilde{V}_0} \cdot J\delta\tilde{V} = \omega_0\left(\frac{\tilde{V}_1}{\tilde{V}_0}\right)\cos(\Delta\omega t - \theta) \quad (38a)$$

$$Q_0\frac{d\theta}{dt} - P_{ex,0}\frac{1}{\tilde{V}_0} \cdot \frac{d\delta\tilde{V}}{dt} + \frac{\omega_0}{\tilde{V}_0} \cdot K\delta\tilde{V} = \omega_0\left(\frac{\tilde{V}_1}{\tilde{V}_0}\right)\sin(\Delta\omega t - \theta) \quad (38b)$$

where P_0 and Q_0 are defined in Section IV, and J and K are defined in (21a) and (21b).

Equations (38a) and (38b) are the basic equations of the free-running oscillator with independent incident signal.

For simplicity, it is assumed that $P_0 \simeq P_{ex,0} \simeq 0$ and $K \simeq 0$. Then, from (38a) and (38b), we have

$$\left(\frac{Q_{ex,0}}{\omega_0}\right) \cdot \frac{d\delta\tilde{V}}{dt} + J\delta\tilde{V} = \tilde{V}_1\cos(\Delta\omega t - \theta) \quad (39a)$$

$$\frac{d\theta}{dt} = \left(\frac{\omega_0}{Q_0}\right)\left(\frac{\tilde{V}_1}{\tilde{V}_0}\right)\sin(\Delta\omega t - \theta). \quad (39b)$$

In order to consider the injection-locking case, it is convenient to introduce a new phase variable $\phi = \Delta\omega t - \theta$ in (39b). Then we obtain Adler's equation [13]. In that case, the total frequency pulling range is determined by

$$2\Delta\omega_L = 2\left(\frac{\omega_0}{Q_0}\right)\left(\frac{\tilde{V}_1}{\tilde{V}_0}\right). \quad (40)$$

Note that Q_0 is not equal to $Q_{ex,0}$, as defined in Section IV. This implies that the effective external Q is affected by the bias circuit.

We are not here concerned with the injection-locking case. Therefore, it can be assumed that $|\Delta\omega| \gg \Delta\omega_L$. In this case, we can express θ in the form

$$\theta = \theta_1 + \delta\theta. \quad (41)$$

Considering $|\delta\theta| \ll 1$,[4] we have

$$d\delta\theta/dt \simeq \Delta\omega_L \sin(\Delta\omega t - \theta_1). \quad (42)$$

Similarly, from (39a), we have

$$\left(\frac{Q_{ex,0}}{\omega_0}\right)\frac{d\delta\tilde{V}}{dt} + J\delta\tilde{V} \simeq V_1\cos(\Delta\omega t - \theta_1). \quad (43)$$

Solving (43) and neglecting the term decaying with time, we obtain

$$\delta\tilde{V} \simeq \frac{(\tilde{V}_1/J)}{1 + \alpha^2}\left\{\cos(\Delta\omega t - \theta_1) + \alpha\sin(\Delta\omega t - \theta_1)\right\} \quad (44)$$

where

$$\alpha = (\Delta\omega Q_{ex,0}/\omega_0 J), \qquad J > 0, \quad Q_{ex,0} > 0.$$

If $|\alpha| \ll 1$ (which is justifiable in the practical device, as previously mentioned), we have

$$\delta\tilde{V} \simeq (\tilde{V}_1/J)\cos(\Delta\omega t - \theta_1). \quad (45)$$

Substituting (42) and (44) into (18), we obtain δV_B, which is the down-converted signal. In the case where Q_{ex} is large to the extent $d\delta\theta/dt$ is negligible, the power dissipation in the IF load G_B is

$$P_I = \frac{G_B}{2\left(G_B + \dfrac{\partial I}{\partial V_B}\right)^2}\left(\frac{\tilde{V}_1}{J}\right)^2\left(\frac{\partial I}{\partial \tilde{V}}\right)^2. \quad (46)$$

The incident RF power is given by

$$P_S = \tfrac{1}{2} Y_0 \tilde{V}_1^2.$$

[4] This is justifiable if $\Delta\omega_L/|\Delta\omega| \ll 1$.

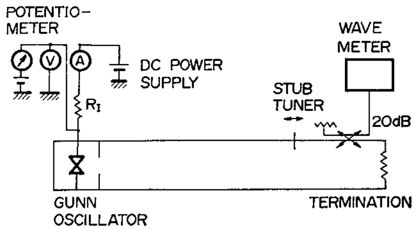

Fig. 3. Simplified experimental setup.

Therefore, conversion gain is given by

$$G_i = \frac{P_I}{P_S} = \frac{G_B}{Y_0\left(G_B + \dfrac{\partial I}{\partial V_B}\right)^2} \cdot \frac{\left(\dfrac{\partial I}{\partial \tilde{V}}\right)^2}{J^2}. \quad (47)$$

From the preceding discussion, it can be seen that (42) and (45) coincide with (26) and (25), respectively, if $\Gamma \tilde{V}_0$ is replaced by $\tilde{V}_1$ of (25) and (26) in the simplified case where $P_0 \simeq P_{ex,0} \simeq 0, K \simeq 0$.

Section VI reports on an investigation by experimenting with a Gunn oscillator wherein conversion gain can be positive, depending on operating conditions.

VI. Experiment

Sections IV-A and V show that the Doppler signal δV_B detected by the oscillator, including the moving object, and the IF signal δV_B, frequency-converted by the free-running oscillator, behave in a similar manner and that their characteristics are independent of the frequency $\Delta\omega$, including the limiting case of $\Delta\omega \simeq 0$. With this understanding, experimental research was conducted for characteristics of the frequency down-conversion using the Gunn oscillator with a movable load.

The setup for the experiment is shown in Fig. 3 (essentially the same as [1, fig. 9]). The output power P_{0f} and the oscillating frequency f_0 of the Gunn oscillator are typically 100 mW and 20.5 GHz, respectively. These values vary with dc bias voltage V_{B_0}. The stub tuner forms a movable load.

The conversion gain obtained is

$$G_i = \frac{\dfrac{1}{2} R_I \left(\dfrac{\Delta I}{2}\right)^2}{P_{0f} |\Gamma|^2}$$

where R_I is the bias resistance, Γ is the reflection coefficient of the movable load, and ΔI is the peak-to-peak value of the sinusoidal fluctuation of the bias current which fluctuates sinusoidally with the movement of the load. Fig. 4 is a plot of the conversion gain for several values of R_I. It can immediately be seen that conversion gain is positive for a large R_I (or small G_I).

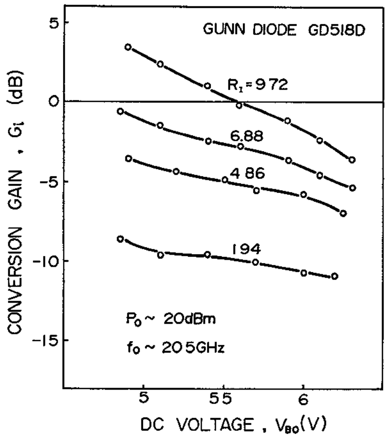

Fig. 4. Conversion gain for a Gunn oscillator (diode: GD518D, NEC).

VII. Conclusion

The theory of the Doppler signal detection with a negative-resistance diode oscillator, operating simultaneously as a signal source and Doppler signal detector, has been developed using a realistic model of the oscillator, including a moving object viewed through an antenna. The effect of the bias circuit taking out the Doppler signal on the RF operation of the oscillator has been taken into account in the theory. Frequency down-conversion with a free-running oscillator was also discussed. The positive conversion gain was demonstrated by the experiment using a Gunn oscillator with the movable load.

Acknowledgment

The author would like to thank S. Nagano and Y. Akaiwa for their useful discussions in the development of this work. He would also like to thank K. Ayaki for his discussion and encouragement and H. Murakami for his encouragement and guidance.

References

[1] S. Nagano and T. Akaiwa, "Behavior of a Gunn diode oscillator with a moving reflector as a self-excited mixer and a load variation detector," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 906–910, Dec. 1971.
[2] S. Mitsui, M. Kotani, and O. Ishihara, "Self-mixing effect of Gunn oscillator," in *Rec. Professional Group on Electron Devices* (IECE Japan), July 1971, Paper ED 71-27.
[3] B. W. Hakki, "GaAs post-threshold microwave amplifier, mixer, and oscillator," *Proc. IEEE* (Lett.), vol. 54, pp. 299–300, Feb. 1966.
[4] W. J. Evans and G. I. Haddad, "Frequency conversion in IMPATT diodes," *IEEE Trans. Electron Devices*, vol. ED-16, pp. 78–87, Jan. 1969.
[5] S. Nagano, H. Ueno, H. Kondo, and H. Murakami, "Self-excited microwave mixer with a Gunn diode and its applications to Doppler radar," *Trans. Inst. Electron. Commun. Eng.* (Japan), vol.52-B, pp. 179–180, Mar. 1969.
[6] "Gunn oscillator is designed for use in 10.69-GHz miniature Doppler radar equipment" (New Product Applications), *IEEE Spectrum*, vol. 7, p. 88, July 1970.
[7] K. Ogiso, T. Nakamura, and K. Shirahata, "Gunn oscillator commonly used as homodyne detector," in *Rec. Professional Group on Microwaves* (IECE Japan), June 1971, Paper MW71-29.
[8] S. Nagano and Y. Akaiwa, "A Doppler radar using a Gunn diode both as a transmitter oscillator and a receiver mixer," in *1971 G-MTT Int. Microwave Symp. Dig.*, pp. 172–173.
[9] D. S. Jones, *The Theory of Electromagnetism.* London, England: Pergamon, 1964, p. 137.
[10] C. Yeh, "Reflection and transmission of electromagnetic waves by a moving dielectric medium," *J. Appl. Phys.*, vol. 36, pp. 3513–3517, Nov. 1965.
[11] K. Kurokawa, "Some basic characteristics of broad-band negative resistance oscillator circuits," *Bell Syst. Tech. J.*, vol. 48, pp. 1937–1955, July–Aug. 1969.
[12] Y. Okabe and S. Okamura, "Analysis of stability and noise of oscillators in free-running, synchronized-running and parallel-running," *Trans. Inst. Electron. Commun. Eng.* (Japan), vol. 52-B, pp. 755–762, Dec. 1969.
[13] R. Adler, "A study of locking phenomena in oscillators," *Proc. IRE*, vol. 34, pp. 351–357, June 1946.